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Volume stabilization and the origin of the inflaton shift symmetry in string theory

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Abstract: One of the main problems of inflation in string theory is finding models with a flat potential while simultaneously stabilizing the volume of the compactified space. This can be achieved in theories where the potential has (an approximate) shift symmetry in the inflaton direction. We will identify a class of models where the shift symmetry uniquely follows from the underlying mathematical structure of the theory. It is related to the symmetry properties of the corresponding coset space and the period matrix of special geometry, which shows how the gauge coupling depends on the volume and the position of the branes. In particular, for type IIB string theory on $K3 \times T^2/\mathbb{Z}_2$ with D3 or D7 moduli belonging to vector multiplets, the shift symmetry is a part of $\mathrm{SO}(2, 2+n)$ symmetry of the coset space $\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,2+n)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n)}$. The absence of a prepotential, specific for the stringy version of supergravity, plays a prominent role in this construction, which may provide a viable mechanism for the accelerated expansion and inflation in the early universe.

Keywords: D-branes, Superstring Vacua, Supergravity Models.

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Contents

1. Introduction

One of the problems of string cosmology is to provide a mechanism for the accelerated expansion and inflation in the early universe, which would be consistent with stabilization of the volume of the compactified space. A possible solution of this problem for the accelerated expansion in a metastable dS space was recently proposed in [1]. However, in order to generalize this solution for the usual slow-roll inflation one would need to find a potential containing a flat direction for the inflaton field. One would also need to make sure that the motion of the field in this potential does not destabilize the volume [2].

It has been suggested in [3] that flat directions for the inflaton field in the D3/D7 brane inflation model [4], consistent with the volume stabilization, can appear as a consequence of shift symmetry with respect to the inflaton field. The existence of this symmetry follows from the assumption of the existence of the BPS state of branes with unbroken supersymmetry. More recently, it was argued in [5] that under the same assumption, namely, the existence of unbroken supersymmetry of the BPS state of branes, the shift symmetry might also appear in the $D3 - \overline{D3}$ inflation model of [2].

The goal of this paper is to show that in a certain class of string theory models the shift symmetry, as well as the existence of the supersymmetric BPS ground state, is not an assumption but an unavoidable consequence of the underlying mathematical structure of the theory. We will give an example of a class of models where this is indeed the case and show that the shift symmetry and the flatness of the inflaton potential in these models are related to the symmetry properties of the corresponding coset space, and the period matrix, which shows how the gauge coupling depends on the volume and the position of the branes.

We will illustrate our general approach investigating the D3/ D7 system [4] of type IIB string theory. We will study it in the context of the special geometry construction [6, 7]. We will be interested here in the setting [3] where the supersymmetry breaking effects, like non-self-dual fluxes on D7 brane, are not yet included. We would like to learn how the stabilization of volume affects the state of the branes, whether their position can still be a modulus. We will see that for D3/ D7 system *one inevitably finds an effective theory with stabilization of the volume and a shift symmetry for the inflaton.* This means that these branes (D3 or D7) move freely in the theory with volume stabilization. A small deviation from the shift symmetry will lead to a slow-roll inflation.

Our starting point is the construction presented by Angelantonj, D'Auria, Ferrara, Trigiante (ADFT) in [6].[1] They have extended the four–dimensional gauged supergravity analysis of type IIB vacua on $K3 \times T^2/\mathbb{Z}_2$ to the case where the D3 and D7 moduli, belonging to $\mathcal{N} = 2$ vector multiplets, are turned on. We will be interested in two cases when either D3 or D7 moduli are present. In each case the overall special geometry corresponds to a symmetric space. We will specify the fundamental shift symmetry for the D7 moduli as a block diagonal symplectic matrix and for the D3 moduli as a lower triangular block form of a symplectic matrix. They will present a part of $\mathrm{SO}(2, 2 + n)$ symmetry of the coset space $\frac{\mathrm{SU}(1,1))}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,2+n)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n)}$. We will identify the gauge couplings in these theories, which will lead us to particular choices of the non-perturbative superpotentials to be used for the stabilization of the volume of the internal space. As the result, in both cases, the potential after volume stabilization respects the inflaton shift symmetry.

1.1 On special Kähler geometry

Let us briefly recall the main formulae of special Kähler geometry [8, 9]. The geometry of the manifold is encoded in the holomorphic section $\Omega = (X^\Lambda, F_\Sigma)$ which, in the *special coordinate* symplectic frame, is expressed in terms of a prepotential $\mathcal{F}(s, t, u, x^k, y^r) = F(X^\Lambda)/(X^0)^2 = \mathcal{F}(X^\Lambda/X^0)$, as follows:

$$\Omega = \left(X^\Lambda,\, F_\Lambda = \frac{\partial F}{\partial X^\Lambda} \right). \tag{1.1}$$

In our case $\mathcal{F}$ is given by eq. (2.2). The Kähler potential K is given by the symplectic invariant expression:

$$K = -\log\left[\mathrm{i}(\overline{X}^\Lambda F_\Lambda - \overline{F}_\Lambda X^\Lambda)\right]. \tag{1.2}$$

Symplectic transformations are realized both on the section Ω as well as on vector fields $(\mathcal{F}^{-\Lambda}_{\mu\nu}, G^{\mu\nu}_{-\Lambda})$:

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}' = \begin{pmatrix} A & -B \\ C & D \end{pmatrix} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}. \tag{1.3}$$

The matrix $\mathcal{S} = \begin{pmatrix} A & -B \\ C & D \end{pmatrix}$ is an $\mathrm{Sp}(2(4 + k), \mathbb{R})$ matrix, with

$$A^T C - C^T A = 0\,, \qquad B^T D - D^T B = 0\,, \qquad A^T D - C^T B = I\,. \tag{1.4}$$

[1] In [7] a model with only D3 moduli and with cosmological applications was proposed. A corrected consistent version of it is closely related to the one in [6].

In terms of K the metric has the form $g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$. The period matrix $\mathcal{N}$ defines the vector kinetic lagrangian as follows:

$$\mathrm{Im}\, \mathcal{F}^{-\Lambda}_{\mu\nu} \overline{\mathcal{N}}_{\Lambda\Sigma} \mathcal{F}^{-\Sigma\mu\nu} = -2\, \mathrm{Im}\, \overline{\mathcal{N}}_{\Lambda\Sigma}\, \mathcal{F}^{\Lambda}_{\mu\nu}\, \mathcal{F}^{\Sigma\mu\nu} + \mathrm{Re}\, \overline{\mathcal{N}}_{\Lambda\Sigma}\, \mathcal{F}^{\Lambda}_{\mu\nu}\, \tilde{\mathcal{F}}^{\Sigma\mu\nu}$$

The period matrix can be introduced via the relations

$$\overline{F_\Lambda} = \overline{\mathcal{N}}_{\Lambda\Sigma} \overline{X^\Sigma}\,, \qquad h_{\Lambda|i} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_i^\Sigma \tag{1.5}$$

$$\overline{\mathcal{N}}_{\Lambda\Sigma} = \hat{h}_{\Lambda|I} \circ (\hat{f}^{-1})^I{}_\Sigma\,, \qquad \text{where} \qquad \hat{f}_I^\Lambda = \begin{pmatrix} \mathcal{D}_i X^\Lambda \\ \overline{X}^\Lambda \end{pmatrix}; \qquad \hat{h}_{\Lambda|I} = \begin{pmatrix} \mathcal{D}_i F_\Lambda \\ \overline{F}_\Lambda \end{pmatrix}. \tag{1.6}$$

The equations of motion of $\mathcal{N} = 2$ supergravity are invariant under symplectic transformations (also the action, except for the kinetic term for the vectors) if the period matrix transforms as follows

$$(\mathcal{N})' = (C + D\mathcal{N})(A + B\mathcal{N})^{-1}\,. \tag{1.7}$$

These transformations are known as dualities. Only part of these symmetries can be realized in perturbation theory as the symmetry of the action: this requires a lower triangular block form of the symplectic matrix with $B = 0$, $D = (A^T)^{-1}$, $A^T C - C^T A = 0$:

$$\mathcal{S}_{\rm pert} = \begin{pmatrix} A & 0 \\ C & (A^T)^{-1} \end{pmatrix}. \tag{1.8}$$

Under these transformations electric fields $\mathcal{F}^\Lambda_{\mu\nu}$ are not mixed with magnetic ones $\mathcal{G}_{\Lambda\mu\nu}$ and X^Λ are not mixed with F_Λ. The lagrangian under such change is invariant up to a surface term

$$\mathcal{L}' = \mathcal{L} + \mathrm{Im}[(C^T A)_{\Lambda\Sigma}\, \mathcal{F}^{-\Lambda}_{\mu\nu}\, \mathcal{F}^{-\Sigma\mu\nu}] = \mathrm{Re}[(C^T A)_{\Lambda\Sigma}]\, \mathcal{F}^{\Lambda}_{\mu\nu}\, \tilde{\mathcal{F}}^{\Sigma\mu\nu}\,. \tag{1.9}$$

It has been discovered in [10] that heterotic string theory requires a version of special geometry which is based on the symplectic section for which the prepotential does not exist. In standard supergravities where the prepotential exist, $X^\Lambda(t^i)$ depend on all independent special coordinates t^i. Here $\Lambda = 0, 1, \ldots, n$ and $i = 1, 2, \ldots, n$. To define the no-prepotential case we have to split all special coordinates into a group $t^i = (t^1 = s, t^a)$ where $a = 2, \ldots, n$. A prepotential only exists when the upper part of the symplectic section can be invertibly mapped to the coordinates of the special Kähler manifold. When this is not the case, i.e. when

$$\frac{\partial X^\Lambda}{\partial s} = 0 \tag{1.10}$$

for one of the coordinates s, then no prepotential exists. However, a completely consistent version of stringy $\mathcal{N} = 2$ supergravity based on the symplectic section is available. In the heterotic theory s is a dilaton-axion superfield, in type IIB it is a volume-4-form superfield. The no-prepotential theory has some particular features which will be used heavily in what follows.

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2. Overview of ADFT construction

In the absence of open-string moduli the four-dimensional $\mathcal{N}=2$ effective supergravity is defined by a special geometry which is described by the coset space $(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)})_s \times (\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)})_t \times (\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)})_u$, where s, t, u denote the scalars of the vector multiplets containing the $K3$-volume and the R-R field on the $K3$, the T^2-complex structure, and the IIB axion-dilaton system, respectively:

$$s = C_{(4)} - \mathrm{i}\,\mathrm{Vol}(K_3), \qquad t = \frac{g_{12}}{g_{22}} + \mathrm{i}\,\frac{\sqrt{\det g}}{g_{22}}\,, \qquad u = C_{(0)} + \mathrm{i}\, e^{\phi}\,, \tag{2.1}$$

where the matrix g denotes the metric on T^2. The total volume of T^2, $\sqrt{\det g}$ belongs to the hypermultiplet and will not be considered here since we will first focus on special geometry.

It was explained in ADFT that the system of interest can be described starting from the following unique trilinear prepotential of special geometry:

$$\mathcal{F}(s,t,u,x^k,y^r) \;=\; stu - \frac{1}{2}\, s\, x^k x^k - \frac{1}{2}\, u\, y^r y^r\,, \tag{2.2}$$

where x^k and y^r are the positions of the D7 and D3-branes along T^2 respectively, $k = 1,\ldots,n_7$, $r = 1,\ldots,n_3$, and summation over repeated indices is understood. The prepotential in eq. (2.2) corresponds to the homogeneous not symmetric spaces called $L(0,n_7,n_3)$ in [11]. If we set either all the x^k or all the y^r to zero, the special geometry in each case describes a symmetric space:

$$\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_s \times \frac{\mathrm{SO}(2,2+n_7)}{\mathrm{SO}(2)\times \mathrm{SO}(2+n_7)}\,, \qquad \text{for } y^r = 0\,, \tag{2.3}$$

$$\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_u \times \frac{\mathrm{SO}(2,2+n_3)}{\mathrm{SO}(2)\times \mathrm{SO}(2+n_3)}\,, \qquad \text{for } x^k = 0\,. \tag{2.4}$$

The components X^Λ, F_Σ of the symplectic section which correctly describe our problem, are chosen by performing a constant symplectic change of basis from the one in (1.1) given in terms of the prepotential in eq. (2.2). The rotated symplectic section is given by

$$\begin{aligned}
X^0 &= \frac{1}{\sqrt{2}}\left(1 - t\,u + \frac{(x^k)^2}{2}\right), & F_0 &= \frac{s\,\left(2 - 2\,t\,u + (x^k)^2\right) + u\,(y^r)^2}{2\sqrt{2}}\,,\\
X^1 &= -\frac{t+u}{\sqrt{2}}\,, & F_1 &= \frac{-2\,s\,(t+u) + (y^r)^2}{2\sqrt{2}}\,,\\
X^2 &= -\frac{1}{\sqrt{2}}\left(1 + t\,u - \frac{(x^k)^2}{2}\right), & F_2 &= \frac{s\,\left(2 + 2\,t\,u - (x^k)^2\right) - u\,(y^r)^2}{2\sqrt{2}}\,,\\
X^3 &= \frac{t-u}{\sqrt{2}}\,, & F_3 &= \frac{2\,s\,(-t+u) + (y^r)^2}{2\sqrt{2}}\,,\\
X^k &= x^k\,, & F_k &= -s\,x^k\,,\\
X^r &= y^r\,, & F_r &= -u\,y^r\,.
\end{aligned} \tag{2.5}$$

The Kähler potential K has the following form:

$$K = -\log\left[-8\,(\mathrm{Im}(s)\;\mathrm{Im}(t)\,\mathrm{Im}(u) - \frac{1}{2}\,\mathrm{Im}(s)\,(\mathrm{Im}(x)^i\,)^2 - \frac{1}{2}\,\mathrm{Im}(u)\,(\mathrm{Im}(y)^r\,)^2)\right], \qquad (2.6)$$

with $\mathrm{Im}(s) < 0$ and $\mathrm{Im}(t)$, $\mathrm{Im}(u) > 0$ at $x^k = y^r = 0$. Note that, since $\partial X^\Lambda/\partial s = 0$ the new sections do not admit a prepotential, and the no-go theorem on partial supersymmetry breaking [12] does not apply in this case.

3. Shift symmetries in positions of D7 branes

The symplectic section and the Kähler potential in absence of D3 moduli are given by

$$\begin{aligned}
X^0 &= \frac{1}{\sqrt{2}}\left(1 - t\,u + \frac{(x^k)^2}{2}\right), & F_0 &= \frac{s\,\left(2 - 2\,t\,u + (x^k)^2\right)}{2\,\sqrt{2}}\,,\\
X^1 &= -\frac{t+u}{\sqrt{2}}\,, & F_1 &= \frac{-2\,s\,\left(t+u\right)}{2\,\sqrt{2}}\,,\\
X^2 &= -\frac{1}{\sqrt{2}}\left(1 + t\,u - \frac{(x^k)^2}{2}\right), & F_2 &= \frac{s\,\left(2 + 2\,t\,u - (x^k)^2\right)}{2\,\sqrt{2}}\,,\\
X^3 &= \frac{t-u}{\sqrt{2}}\,, & F_3 &= \frac{2\,s\,\left(-t+u\right)}{2\,\sqrt{2}}\,,\\
X^k &= x^k\,, & F_k &= -s\,x^k\,.
\end{aligned} \qquad (3.1)$$

$$K = -\log\left[-8\;\mathrm{Im}(s)\right] - \log\left[\mathrm{Im}(t)\mathrm{Im}(u) + \frac{1}{16}\,(\mathrm{Im}(x)^i\,)^2\right]. \qquad (3.2)$$

The $\mathrm{SO}(2, 2+n_7)$ symmetry is realized manifestly. It is useful to switch to light-cone variables, $X^\mp = \frac{X^0 \mp X^2}{\sqrt{2}}$ and $Y^\mp = \frac{X^1 \mp X^3}{\sqrt{2}}$, where

$$X^-X^+ + X^+X^- + Y^-Y^+ + Y^+Y^- - (X^k)^2 = 0\,. \qquad (3.3)$$

In this basis

$$X^\Lambda = \{X^- = 1,\, X^+ = -tu + x^2/2,\, Y^- = -t,\, Y^+ = -u,\, X^k = x^k\} \qquad (3.4)$$

$$F_\Lambda = \left\{F^X_- = s\left(-tu + \frac{x^2}{2}\right),\, F^X_+ = s,\, F^Y_- = -su,\, F^Y_+ = -st,\, F_k = -sx^k\right\}. \qquad (3.5)$$

Thus the model is defined by

$$X^\Lambda \eta_{\Lambda\Sigma} X^\Sigma = 0 \qquad F_\Lambda = s\eta_{\Lambda\Sigma}X^\Lambda\,. \qquad (3.6)$$

Where indices are raised and lowered using η

$$\eta = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \qquad (3.7)$$

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The shift of the D7 moduli with the real parameters α^k is given by,

$$(x^k)' = x^k + \alpha^k \qquad s' = s \qquad t' = t \qquad u' = u \tag{3.8}$$

can be realized as a perturbative symplectic transformation, a block-diagonal matrix

$$\mathcal{S}^{D7} = \begin{pmatrix} A & 0 \\ 0 & (A^T)^{-1} \end{pmatrix}$$

with $B = C = 0$ and $D = (A^T)^{-1}$. For the simple case of one D7 brane, $k = 1$, $A^\Lambda{}_\Sigma$ is equal to

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha^2/2 & 1 & 0 & 0 & \alpha \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.9}$$

so that the shift is defined by $(X^\Lambda)' = A^\Lambda{}_\Sigma X^\Sigma$. In this model one can write down the period matrix, as was shown in [10]

$$\mathcal{N}_{\Lambda\Sigma}(X) = (s - \bar{s})(\Phi_\Lambda \bar{\Phi}_\Sigma + \bar{\Phi}_\Lambda \Phi_\Sigma) + \bar{s}\eta_{\Lambda\Sigma} \ ,$$

where $\Phi_\Lambda = \eta_{\Lambda\Sigma}\Phi^\Sigma$, where one can raise or lower indices with η and $\Phi^\Lambda = \frac{X^\Lambda}{\sqrt{X^\Sigma \eta_{\Sigma\Pi} \bar{X}^\Pi}}$. The real part of the period matrix is

$$\mathrm{Re}\, \mathcal{N}_{\Lambda\Sigma}(X) = (s + \bar{s})\eta_{\Lambda\Sigma} \, .$$

This gives us an information on the gauge coupling for the vector fields. In particular, the axion from the s field, the $C_{(4)}$, couples to $FF*$. It cannot be shifted by a holomorphic function of x^2 which would be required, as shown in [3] to change the Kähler potential from $(\mathrm{Im}\, x)^2$ to $x\bar{x}$-type.

In the sector of the gauge fields where the abelian gauge symmetry is enhanced to a non-abelian symmetry, instanton corrections may lead to terms in the action of the form $e^{-1/g^2_{YM}}$. This in turn may be understood as coming from a holomorphic superpotential $W = e^{-as}$, where a is some constant. Notice that the definition of the volume-4-form superfield s is such that the Kähler potential has a shift symmetry under the translation of the D7 brane position, i.e. it depends on $(\mathrm{Im}\, x)^2$. The analysis of the special geometry shows that the non-perturbative potential for the KKLT stabilization can depend on e^{-as} but cannot depend on any holomorphic function of x as it follows from the period matrix $\mathcal{N}$. This accomplishes the proof of the shift symmetry for the motion of the D7 brane, both in the Kähler potential and in the superpotential.

4. Shift symmetries in positions of D3 branes

The symplectic section and the Kähler potential in absence of D7 moduli are given by

$$\begin{aligned} X^- &= 1\,, & F^X_- &= -stu + \frac{u\,(y^r)^2}{2}\,,\\ X^+ &= -tu\,, & F^X_+ &= s\,,\\ Y^- &= -t & F^Y_- &= -su\,,\\ Y^+ &= -u & F^Y_+ &= -st + \frac{(y^r)^2}{2}\,,\\ X^r &= y^r & F_r &= -u\,y^r\,. \end{aligned} \tag{4.1}$$

$$K = -\log[-8\,\mathrm{Im}(u)] - \log\left[\mathrm{Im}(s)\,\mathrm{Im}(t) + \frac{1}{16}\,(\mathrm{Im}(y)^r\,)^2\right]. \tag{4.2}$$

The shift in D3 position with real parameters β^r,

$$(y^r)' = y^r + \beta^r \qquad s' = s \qquad t' = t \qquad u' = u \tag{4.3}$$

can be realized as a lower triangular block form symplectic matrix:

$$\mathcal{S}^{D3} = \begin{pmatrix} A & 0 \\ C & (A^T)^{-1} \end{pmatrix}.$$

In case of one D3 brane, $r = 1$ we find for $A^\Lambda{}_\Sigma$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \beta & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.4}$$

and for $C_{\Lambda\Sigma}$

$$C = \begin{pmatrix} 0 & 0 & 0 & -\beta^2/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta^2/2 & 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & \beta & 0 \end{pmatrix}. \tag{4.5}$$

One can calculate the period matrix and find out the gauge couplings for this case. Alternatively, one can study its symmetry properties using eq. (1.7), adapted to our case. To simplify things we may look at the imaginary part of the period matrix, Im $\mathcal{N}$.

$$(\mathrm{Im}\,\mathcal{N})' = (A^T)^{-1}\,\mathrm{Im}\,\mathcal{N} A^{-1}\,. \tag{4.6}$$

The crucial observation here is that the total period matrix $\mathcal{N}_{\Lambda\Sigma}$ can be split into the part which has terms which interact with the graviphoton vector field and those which are decoupled. In our basis this is a split into $\mathcal{N}_{--}$, $\mathcal{N}_{-i}$ and $\mathcal{N}_{ij}$. Here i includes all components but $-$. Using the explicit form of the matrix A (4.4) in eq. (4.6) one can establish that only the components $\mathcal{N}_{--}$, $\mathcal{N}_{-i}$ transform, all components $\mathcal{N}_{ij}$ are invariant under the shift. The non-perturbative instanton superpotential depends only on gauge coupling in the ij sector of the theory, graviphoton vector field does not contribute.[2] Thus we conclude that the superpotential must be shift invariant when we use special coordinates for which Kähler potential was invariant from the very beginning.

5. Conclusion

The bottom line of this strict special geometry analysis for cosmological applications is the following. Assuming that the volume of T^2 is the same as the one for $K3$, and that the dilaton and complex structure are fixed via fluxes as in [13], one finds the Kähler potential used in this paper. Using the notation of [3] one finds $K = -3\log[(\rho+\bar\rho-(\phi+\bar\phi)^2]$ for D3 and $K = -3\log[(\rho+\bar\rho)] + (S+\bar S)^2/2$ for D7. Also one may follow ADFT [6] and introduce the gaugings which represents turning on fluxes in string theory. The gaugings lead to a non-vanishing potentials, which may break $\mathcal{N}=2$ down to $\mathcal{N}=1$ and stabilize the axion-dilaton and the complex structure. The gauging (introduction of fluxes) does not affect the period matrix of vector multiplets. Therefore, as shown above, the non-perturbative superpotential which may be used for stabilization must be shift-invariant and is given by $W = W_0 + Ae^{-a\rho}$ as in [1, 3]. This is valid for the case when either D3 is light and D7 is heavy and only D3 can move or D7 is light and D3 is heavy and only D7 can move. In both cases the position of the moving brane is a modulus, since both the Kähler potential and the superpotential are shift invariant.

It is amazing that the proof of the shift symmetry for the application to cosmology is based on exactly the same set of special geometry tools (see eqs. (1.1)–(1.3)), which were used earlier in studies of extremal black holes: they explained the attractor behavior of the scalars near the black hole horizon and the duality symmetries of the black hole entropy formula.

In the setting of this paper special geometry controls duality symmetries which include a shift symmetry for the inflaton field. This may provide a fundamental basis for the realistic string cosmology where almost flat potentials are required for explanation of the cosmological observations.

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A. Shift symmetries of homogeneous non-symmetric space $L(0, n_7, n_3)$

Here we consider the general case when both positions of D7 as well as positions of D3

[2] At the point of enhancement of gauge symmetry where the non-abelian gauge symmetry arises, the graviphoton does not contribute by symmetry reasons. We are grateful to T. Banks for explaining this.

branes are turned on. The section of the $Sp(2(4+n_7+n_3),R)$ bundle is

$$X = \begin{pmatrix} X^- = 1 \\ X^+ = -tu + x^2/2 \\ Y^- = -t \\ Y^+ = -u \\ X^k = x^k \\ X^r = y^r \end{pmatrix} \tag{A.1}$$

$$F = \begin{pmatrix} F_-^X = s(-tu + x^2/2) + uy^2/2 \\ F_+^X = s \\ F_-^Y = -su \\ F_+^Y = -st + y^2/2 \\ F_k = -sx^k \\ F_r = -uy^r \end{pmatrix}. \tag{A.2}$$

Under $x^k \to x^k + \alpha^k$ and $y^r \to y^r + \beta^r$,

$$\Delta X = \begin{pmatrix} 0 \\ \alpha^k X^k + \alpha^2 X^-/2 \\ 0 \\ 0 \\ \alpha X^- \\ \beta X^- \end{pmatrix} \tag{A.3}$$

$$\Delta F = \begin{pmatrix} -\alpha^k F_k + \alpha^2 F_+^X/2 - \beta^r F_r - \beta^2 Y^+/2 \\ 0 \\ 0 \\ \beta^r X^r + \beta^2 X^-/2 \\ -\alpha F_+^X \\ \beta Y^+ \end{pmatrix}. \tag{A.4}$$

Note that ΔF has contributions from both X and F, whereas ΔX involves only X. This implies that B in the transformation matrix above is zero. We can reconstruct A, C and D by noting that $X + \Delta X = AX$ and $F + \Delta F = CX + DF$. Using this, we get that,

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \alpha^2/2 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \alpha & 0 & 0 & 0 & 1 & 0 \\ \beta & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{A.5}$$

$$D = \begin{pmatrix} 1 & \alpha^2/2 & 0 & 0 & -\alpha & -\beta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\alpha & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{A.6}$$

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$$C = \begin{pmatrix} 0 & 0 & 0 & -\beta^2/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \beta^2/2 & 0 & 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 \end{pmatrix} . \tag{A.7}$$

We need to check that the 10x10 matrix is symplectic which amounts to the conditions $A^T C = (A^T C)^T$ and $A^T D = I$. This is in fact true so the transformation is a symplectic one. As far as the period matrix is concerned, we need to know only the properties of the gauge fields kinetic matrix of the form $f(z, \bar{z})\eta_{AB}$ where the graviphoton has to be excluded. This will provide us the information on the kinetic terms for the vector fields with the enhancement of gauge symmetries to the non-abelian ones.

$$\mathrm{Im}\,\mathcal{N}'_{\Lambda\Sigma} = \mathrm{Im}\,f(D\eta D^T)_{\Lambda\Sigma} . \tag{A.8}$$

An explicit calculation gives for $D\eta D^T$

$$D\eta D^T = \begin{pmatrix} \beta^2 & 1 & 0 & 0 & \beta \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \beta & 0 & 0 & -1 \end{pmatrix} \tag{A.9}$$

which proves that $f' = f$. Therefore the non-perturbative superpotential is shift symmetric.

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